

What is claimed is:

1 1. An apparatus for use in encryption or decryption, for
2 solving a system of linear equations $Ax=b$ in n unknowns on a
3 finite field $GF(p)$, where p is a prime, n is a positive integer,
4 A is a coefficient matrix consisting of elements of n rows and n
5 columns, x is a vector of unknowns consisting of n elements, and
6 b is a constant vector consisting of n elements, the apparatus
7 comprising:

8 parameter storing means for storing the coefficient matrix
9 A and the constant vector b ;

10 triangular transforming means for reading the coefficient
11 matrix A and the constant vector b from the parameter storing
12 means, and transforming the read coefficient matrix A and
13 constant vector b to generate a coefficient matrix C and a
14 constant vector d for a system of linear equations $Cx=d$ in n
15 unknowns that is equivalent to the system of linear equations
16 $Ax=b$, the coefficient matrix C consisting of elements of n rows
17 and n columns and the constant vector d consisting of n elements,
18 wherein the coefficient matrix A is triangular transformed into
19 the coefficient matrix C of upper triangular form without
20 diagonal elements of the coefficient matrix A being changed to
21 1;

22 diagonal element inverting means for calculating inverses of
23 diagonal elements of the generated coefficient matrix C on the

finite field $GF(p)$; and

equation computing means for solving the system of linear equations $Cx=d$ using the coefficient matrix C , the constant vector d , and the inverses of the diagonal elements of the coefficient matrix C , to thereby solve the system of linear equations $Ax=b$.

2. The apparatus of Claim 1,

wherein the triangular transforming means performs one or more successive transformation processes to generate the coefficient matrix C and the constant vector d of the system of linear equations $Cx=d$ from the coefficient matrix A and the constant vector b of the system of linear equations $Ax=b$,

wherein in each transformation process the triangular transforming means transforms a coefficient matrix and a constant vector of a system of linear equations in n unknowns, into a coefficient matrix and a constant vector of a system of linear equations in n unknowns that is equivalent to the system of linear equations before the transformation, where the system of linear equations $Ax=b$ is subjected to the first transformation process and the system of linear equations $Cx=d$ is generated as a result of the last transformation process,

wherein in each transformation process the system of linear equations in n unknowns that is subjected to the transformation

includes one pivotal equation which is a linear equation serving as a pivot for the transformation and one or more object equations which are linear equations to be transformed, and the triangular transforming means transforms each of the object equations into an equation equivalent to the object equation by defining a first coefficient group containing at least one value related to the pivotal equation and a second coefficient group containing $n+1$ values related to the pivotal equation, changing a nonzero coefficient in the object equation to 0, and multiplying each of a constant and n coefficients in the object equation by the value in the first coefficient group, and subtracting the $n+1$ values in the second coefficient group respectively from the $n+1$ multiplication results.

3. The apparatus of Claim 2,

wherein each transformation process has transformation subprocesses each for transforming a separate one of the object equations,

wherein in each transformation subprocess the triangular transforming means

(a) chooses a nonzero coefficient from the pivotal equation and sets the chosen nonzero coefficient into the first coefficient group,

10 (b) chooses a nonzero coefficient from the object equation,
11 multiplies each of a constant and n coefficients in the pivotal
12 equation by the nonzero coefficient chosen from the object
13 equation, and sets $n+1$ values obtained by the multiplications
14 into the second coefficient group,

15 (c) changes the chosen nonzero coefficient in the object
16 equation to 0, and

17 (d) multiplies each of a constant and n coefficients in the
18 object equation by the nonzero coefficient in the first
19 coefficient group, and subtracts the $n+1$ values in the second
20 coefficient group respectively from the $n+1$ multiplication
21 results.

22 4. The apparatus of Claim 3,

23 wherein when the diagonal elements of the coefficient matrix
24 C are denoted by m_i ($i=1,2,\dots,n$) and the inverses of the
25 diagonal elements m_i ($i=1,2,\dots,n$) in the finite field $GF(p)$ are
26 denoted by I_i ($i=1,2,\dots,n$), the diagonal element inverting means
27 includes

28 (a) a multiplying unit for computing

$$29 \quad t_i = \prod_{k=1}^n m_k \cdot (\text{except } m_i) \bmod p \quad (i=1,2,\dots,n)$$

30 and

$$31 \quad t = \prod_{k=1}^n m_k \bmod p$$

12 (b) a first inverting unit for computing
 13 $u=1/t \bmod p$
 14 and
 15 (c) a second inverting unit for computing
 16 $I_i=u \times t_i \bmod p \ (i=1,2,\dots,n)$
 17 to find the inverses $I_i \ (i=1,2,\dots,n)$.

1 5. The apparatus of Claim 4,
 2 wherein the multiplying unit calculates

$$\begin{aligned} s_1 &= m_1 \times m_2 \bmod p \\ s_2 &= s_1 \times m_3 \bmod p \\ &\vdots \\ s_{n-3} &= s_{n-4} \times m_{n-2} \bmod p \end{aligned}$$

in the stated order, then calculates

$$\begin{aligned} t_n &= s_{n-3} \times m_{n-1} \bmod p \\ t_{n-1} &= s_{n-3} \times m_n \bmod p \\ s_n &= m_{n-1} \times m_n \bmod p \\ t_{n-2} &= s_{n-4} \times s_n \bmod p \\ s_{n-1} &= m_{n-2} \times s_n \bmod p \\ t_{n-3} &= s_{n-5} \times s_{n-1} \bmod p \\ s_{n-2} &= m_{n-3} \times s_{n-1} \bmod p \\ t_{n-4} &= s_{n-6} \times s_{n-2} \bmod p \\ &\vdots \\ s_5 &= m_4 \times s_6 \bmod p \end{aligned}$$

18 $t_3 = s_1 \times s_5 \bmod p$
 19 $s_4 = m_3 \times s_5 \bmod p$
 20 $t_2 = m_1 \times s_4 \bmod p$
 21 $t_1 = m_2 \times s_4 \bmod p$
 22 in the stated order, and lastly calculates
 23 $t = t_j \times m_j$
 24 for a value j chosen from a set of positive integers
 25 $\{1, 2, \dots, n\}$.

6. The apparatus of Claim 2,

wherein each transformation process has a coefficient group calculation process and transformation subprocesses, performed following the coefficient group calculation process, each for transforming a separate one of the object equations,

wherein in the coefficient group calculation process the triangular transforming means

(a) chooses m nonzero coefficients by taking one nonzero coefficient from each of the pivotal equation and the object equations, multiplies each combination of $(m-1)$ of the chosen nonzero coefficients, and sets the m multiplication results into the first coefficient group, m being a positive integer no smaller than 2, and

(b) multiplies each of a constant and n coefficients in the pivotal equation by a multiplication result in the first

coefficient group for a combination of nonzero coefficients that does not include a nonzero coefficient chosen from the pivotal equation, and sets $n+1$ values obtained by the multiplications into the second coefficient group, and

wherein in each of the transformation subprocesses following the coefficient group calculation process, the triangular transforming means

(a) changes a nonzero coefficient chosen from the object equation in the coefficient group calculation process, to 0 in the object equation, and

(b) multiplies each of a constant and n coefficients in the object equation by a multiplication result in the first coefficient group for a combination of nonzero coefficients that does not include the nonzero coefficient chosen from the object equation, and subtracts the $n+1$ values in the second coefficient group respectively from the $n+1$ multiplication results.

7. The apparatus of Claim 6,

wherein when the diagonal elements of the coefficient matrix C are denoted by m_i ($i=1,2,\dots,n$) and the inverses of the diagonal elements m_i ($i=1,2,\dots,n$) in the finite field $GF(p)$ are denoted by I_i ($i=1,2,\dots,n$), the diagonal element inverting means includes

(a) a multiplying unit for computing

$$t_i = \prod_{k=1}^n m_k \text{ (except } m_i) \text{ mod } p \text{ (} i=1,2,\dots,n)$$

and

$$t = \prod_{k=1}^n m_k \text{ mod } p$$

(b) a first inverting unit for computing

$$u = 1/t \text{ mod } p$$

and

(c) a second inverting unit for computing

$$I_i = u \times t_i \text{ mod } p \text{ (} i=1,2,\dots,n)$$

to find the inverses I_i ($i=1,2,\dots,n$).

8. The apparatus of Claim 7,

wherein the multiplying unit calculates

$$s_1 = m_1 \times m_2 \text{ mod } p$$

$$s_2 = s_1 \times m_3 \text{ mod } p$$

:

$$s_{n-3} = s_{n-4} \times m_{n-2} \text{ mod } p$$

in the stated order, then calculates

$$t_n = s_{n-3} \times m_{n-1} \text{ mod } p$$

$$t_{n-1} = s_{n-3} \times m_n \text{ mod } p$$

$$s_n = m_{n-1} \times m_n \text{ mod } p$$

$$t_{n-2} = s_{n-4} \times s_n \text{ mod } p$$

$$s_{n-1} = m_{n-2} \times s_n \text{ mod } p$$

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$$\begin{aligned}
 t_{n-3} &= s_{n-5} \times s_{n-1} \mod p \\
 s_{n-2} &= m_{n-3} \times s_{n-1} \mod p \\
 t_{n-4} &= s_{n-6} \times s_{n-2} \mod p \\
 &\vdots \\
 s_5 &= m_4 \times s_6 \mod p \\
 t_3 &= s_1 \times s_5 \mod p \\
 s_4 &= m_3 \times s_5 \mod p \\
 t_2 &= m_1 \times s_4 \mod p \\
 t_1 &= m_2 \times s_4 \mod p
 \end{aligned}$$

in the stated order, and lastly calculates

$$t = t_j \times m_j$$

for a value j chosen from a set of positive integers $\{1, 2, \dots, n\}$.

9. An apparatus for use in encryption or decryption, for computing an inverse I of an element y in $GF(q)$ which is an extension field of a finite field $GF(p)$, where p is a prime, $q = p^n$, and n is a positive integer, the apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax = b$ in n unknowns, using the element y and all coefficients of a generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system of linear equations $Ax = b$, the equation solving means including

the apparatus of Claim 1; and

inverse computing means for computing the inverse I using the root α and the solutions found by the equation solving means.

10. An apparatus for use in encryption or decryption, for computing an inverse I of an element y in $GF(q)$ which is an extension field of a finite field $GF(p)$, where p is a prime, $q=p^n$, and n is a positive integer, the apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax=b$ in n unknowns, using the element y and all coefficients of a generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system of linear equations $Ax=b$, the equation solving means including the apparatus of Claim 2; and

inverse computing means for computing the inverse I using the root α and the solutions found by the equation solving means.

11. An apparatus for use in encryption or decryption, for computing an inverse I of an element y in $GF(q)$ which is an extension field of a finite field $GF(p)$, where p is a prime, $q=p^n$, and n is a positive integer, the apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax=b$

7 in n unknowns, using the element y and all coefficients of a
8 generator polynomial of $GF(q)$ whose root is α ;

9 equation solving means for finding solutions of the system
10 of linear equations $Ax=b$, the equation solving means including
11 the apparatus of Claim 3; and

12 inverse computing means for computing the inverse I using the
13 root α and the solutions found by the equation solving means.

1 12. An apparatus for use in encryption or decryption, for
2 computing an inverse I of an element y in $GF(q)$ which is an
3 extension field of a finite field $GF(p)$, where p is a prime,
4 $q=p^n$, and n is a positive integer, the apparatus comprising:

5 equation generating means for generating a coefficient matrix
6 A and a constant vector b for a system of linear equations $Ax=b$
7 in n unknowns, using the element y and all coefficients of a
8 generator polynomial of $GF(q)$ whose root is α ;

9 equation solving means for finding solutions of the system
10 of linear equations $Ax=b$, the equation solving means including
11 the apparatus of Claim 4; and

12 inverse computing means for computing the inverse I using the
13 root α and the solutions found by the equation solving means.

1 13. An apparatus for use in encryption or decryption, for
2 computing an inverse I of an element y in $GF(q)$ which is an

3 extension field of a finite field $GF(p)$, where p is a prime,
4 $q=p^n$, and n is a positive integer, the apparatus comprising:

5 equation generating means for generating a coefficient matrix
6 A and a constant vector b for a system of linear equations $Ax=b$
7 in n unknowns, using the element y and all coefficients of a
8 generator polynomial of $GF(q)$ whose root is α ;

9 equation solving means for finding solutions of the system
10 of linear equations $Ax=b$, the equation solving means including
11 the apparatus of Claim 5; and

12 inverse computing means for computing the inverse I using the
13 root α and the solutions found by the equation solving means.

14. An apparatus for use in encryption or decryption, for
15 computing an inverse I of an element y in $GF(q)$ which is an
16 extension field of a finite field $GF(p)$, where p is a prime,
17 $q=p^n$, and n is a positive integer, the apparatus comprising:

18 equation generating means for generating a coefficient matrix
19 A and a constant vector b for a system of linear equations $Ax=b$
20 in n unknowns, using the element y and all coefficients of a
21 generator polynomial of $GF(q)$ whose root is α ;

22 equation solving means for finding solutions of the system
23 of linear equations $Ax=b$, the equation solving means including
24 the apparatus of Claim 6; and

25 inverse computing means for computing the inverse I using the

13 root α and the solutions found by the equation solving means.

1 15. An apparatus for use in encryption or decryption, for
2 computing an inverse I of an element y in $GF(q)$ which is an
3 extension field of a finite field $GF(p)$, where p is a prime,
4 $q=p^n$, and n is a positive integer, the apparatus comprising:

5 equation generating means for generating a coefficient matrix
6 A and a constant vector b for a system of linear equations $Ax=b$
7 in n unknowns, using the element y and all coefficients of a
8 generator polynomial of $GF(q)$ whose root is α ;

9 equation solving means for finding solutions of the system
10 of linear equations $Ax=b$, the equation solving means including
11 the apparatus of Claim 7; and

12 inverse computing means for computing the inverse I using the
13 root α and the solutions found by the equation solving means.

1 16. An apparatus for use in encryption or decryption, for
2 computing an inverse I of an element y in $GF(q)$ which is an
3 extension field of a finite field $GF(p)$, where p is a prime,
4 $q=p^n$, and n is a positive integer, the apparatus comprising:

5 equation generating means for generating a coefficient matrix
6 A and a constant vector b for a system of linear equations $Ax=b$
7 in n unknowns, using the element y and all coefficients of a
8 generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system of linear equations $Ax=b$, the equation solving means including the apparatus of Claim 8; and

inverse computing means for computing the inverse I using the root α and the solutions found by the equation solving means.

17. A record medium reproducing apparatus for computing, when copyrighted digital content has been encrypted using a discrete logarithm problem on an elliptic curve E over $GF(q)$ as a basis for security and recorded on a record medium, an inverse I of an element y in $GF(q)$ to decrypt the encrypted digital content recorded on the record medium, where $GF(q)$ is an extension field of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive integer, and G is a base point of the elliptic curve E , the record medium reproducing apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax=b$ in n unknowns, using the element y and all coefficients of a generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system of linear equations $Ax=b$, the equation solving means including the apparatus of Claim 1; and

inverse computing means for computing the inverse I using the root α and the solutions found by the equation solving means.

1 18. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an
5 element y in $GF(q)$ to decrypt the encrypted digital content
6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

10 equation generating means for generating a coefficient matrix
11 A and a constant vector b for a system of linear equations $Ax=b$
12 in n unknowns, using the element y and all coefficients of a
13 generator polynomial of $GF(q)$ whose root is α ;

14 equation solving means for finding solutions of the system
15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 2; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

1 19. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an

5 element y in $GF(q)$ to decrypt the encrypted digital content
6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

10 equation generating means for generating a coefficient matrix
11 A and a constant vector b for a system of linear equations $Ax=b$
12 in n unknowns, using the element y and all coefficients of a
13 generator polynomial of $GF(q)$ whose root is α ;

14 equation solving means for finding solutions of the system
15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 3; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

20. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an
5 element y in $GF(q)$ to decrypt the encrypted digital content
6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax=b$ in n unknowns, using the element y and all coefficients of a generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system of linear equations $Ax=b$, the equation solving means including the apparatus of Claim 4; and

inverse computing means for computing the inverse I using the root α and the solutions found by the equation solving means.

21. A record medium reproducing apparatus for computing, when copyrighted digital content has been encrypted using a discrete logarithm problem on an elliptic curve E over $GF(q)$ as a basis for security and recorded on a record medium, an inverse I of an element y in $GF(q)$ to decrypt the encrypted digital content recorded on the record medium, where $GF(q)$ is an extension field of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive integer, and G is a base point of the elliptic curve E , the record medium reproducing apparatus comprising:

equation generating means for generating a coefficient matrix A and a constant vector b for a system of linear equations $Ax=b$ in n unknowns, using the element y and all coefficients of a generator polynomial of $GF(q)$ whose root is α ;

equation solving means for finding solutions of the system

15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 5; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

1 22. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an
5 element y in $GF(q)$ to decrypt the encrypted digital content
6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

10 equation generating means for generating a coefficient matrix
11 A and a constant vector b for a system of linear equations $Ax=b$
12 in n unknowns, using the element y and all coefficients of a
13 generator polynomial of $GF(q)$ whose root is α ;

14 equation solving means for finding solutions of the system
15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 6; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

1 23. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an
5 element y in $GF(q)$ to decrypt the encrypted digital content
6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

10 equation generating means for generating a coefficient matrix
11 A and a constant vector b for a system of linear equations $Ax=b$
12 in n unknowns, using the element y and all coefficients of a
13 generator polynomial of $GF(q)$ whose root is α ;

14 equation solving means for finding solutions of the system
15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 7; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

1 24. A record medium reproducing apparatus for computing, when
2 copyrighted digital content has been encrypted using a discrete
3 logarithm problem on an elliptic curve E over $GF(q)$ as a basis
4 for security and recorded on a record medium, an inverse I of an
5 element y in $GF(q)$ to decrypt the encrypted digital content

6 recorded on the record medium, where $GF(q)$ is an extension field
7 of a finite field $GF(p)$, p is a prime, $q=p^n$, n is a positive
8 integer, and G is a base point of the elliptic curve E , the
9 record medium reproducing apparatus comprising:

10 equation generating means for generating a coefficient matrix
11 A and a constant vector b for a system of linear equations $Ax=b$
12 in n unknowns, using the element y and all coefficients of a
13 generator polynomial of $GF(q)$ whose root is α ;

14 equation solving means for finding solutions of the system
15 of linear equations $Ax=b$, the equation solving means including
16 the apparatus of Claim 8; and

17 inverse computing means for computing the inverse I using the
18 root α and the solutions found by the equation solving means.

25. A method for solving a system of linear equations $Ax=b$
in n unknowns on a finite field $GF(p)$ where p is a prime, n is a
positive integer, A is a coefficient matrix consisting of
elements of n rows and n columns, x is a vector of unknowns
consisting of n elements, and b is a constant vector consisting
of n elements, for use in encryption or decryption in an
apparatus equipped with parameter storing means which stores the
coefficient matrix A and the constant vector b , the method
comprising:

11 a triangular transforming step for reading the coefficient
12 matrix A and the constant vector b from the parameter storing
13 means, and transforming the read coefficient matrix A and
14 constant vector b to generate a coefficient matrix C and a
15 constant vector d for a system of linear equations $Cx=d$ in n
16 unknowns that is equivalent to the system of linear equations
17 $Ax=b$, the coefficient matrix C consisting of elements of n rows
18 and n columns and the constant vector d consisting of n elements,
19 wherein the coefficient matrix A is triangular transformed into
20 the coefficient matrix C of upper triangular form without
21 diagonal elements of the coefficient matrix A being changed to
22 1;

23 a diagonal element inverting step for calculating inverses
24 of diagonal elements of the generated coefficient matrix C on the
25 finite field $GF(p)$; and

26 an equation computing step for solving the system of linear
27 equations $Cx=d$ using the coefficient matrix C , the constant
28 vector d , and the inverses of the diagonal elements of the
29 coefficient matrix C , to thereby solve the system of linear
30 equations $Ax=b$.

1 26. The method of Claim 25,

2 wherein the triangular transforming step includes one or more
3 successive transformation processes to generate the coefficient

4 matrix C and the constant vector d of the system of linear
5 equations $Cx=d$ from the coefficient matrix A and the constant
6 vector b of the system of linear equations $Ax=b$,

7 wherein in each transformation process a coefficient matrix
8 and a constant vector of a system of linear equations in n
9 unknowns are transformed into a coefficient matrix and a constant
10 vector of a system of linear equations in n unknowns that is
11 equivalent to the system of linear equations before the
12 transformation, where the system of linear equations $Ax=b$ is
13 subjected to the first transformation process and the system of
14 linear equations $Cx=d$ is generated as a result of the last
15 transformation process,

16 wherein in each transformation process the system of linear
17 equations in n unknowns that is subjected to the transformation
18 includes one pivotal equation which is a linear equation serving
19 as a pivot for the transformation and one or more object
20 equations which are linear equations to be transformed, and each
21 of the object equations is transformed into an equation
22 equivalent to the object equation by

23 defining a first coefficient group containing at least one
24 value related to the pivotal equation and a second coefficient
25 group containing $n+1$ values related to the pivotal equation,

26 changing a nonzero coefficient in the object equation to 0,
27 and

28 multiplying each of a constant and n coefficients in the
29 object equation by the value in the first coefficient group, and
30 subtracting the $n+1$ values in the second coefficient group
31 respectively from the $n+1$ multiplication results.

1 27. The method of Claim 26,
2 wherein each transformation process has transformation
3 subprocesses each for transforming a separate one of the object
4 equations,

5 wherein in each transformation subprocess

6 (a) a nonzero coefficient is chosen from the pivotal equation
7 and set into the first coefficient group,

8 (b) a nonzero coefficient is chosen from the object equation,
9 each of a constant and n coefficients in the pivotal equation is
10 multiplied by the nonzero coefficient chosen from the object
11 equation, and $n+1$ values obtained by the multiplications are set
12 into the second coefficient group,

13 (c) the chosen nonzero coefficient in the object equation is
14 changed to 0, and

15 (d) each of a constant and n coefficients in the object
16 equation is multiplied by the nonzero coefficient in the first
17 coefficient group, and the $n+1$ values in the second coefficient
18 group are subtracted respectively from the $n+1$ multiplication
19 results.

$$s_{n-3}=s_{n-4} \times m_{n-2} \bmod p$$

in the stated order, then calculates

$$t_n=s_{n-3} \times m_{n-1} \bmod p$$

$$t_{n-1}=s_{n-3} \times m_n \bmod p$$

$$s_n=m_{n-1} \times m_n \bmod p$$

$$t_{n-2}=s_{n-4} \times s_n \bmod p$$

$$s_{n-1}=m_{n-2} \times s_n \bmod p$$

$$t_{n-3}=s_{n-5} \times s_{n-1} \bmod p$$

$$s_{n-2}=m_{n-3} \times s_{n-1} \bmod p$$

$$t_{n-4}=s_{n-6} \times s_{n-2} \bmod p$$

:

$$s_5=m_4 \times s_6 \bmod p$$

$$t_3=s_1 \times s_5 \bmod p$$

$$s_4=m_3 \times s_5 \bmod p$$

$$t_2=m_1 \times s_4 \bmod p$$

$$t_1=m_2 \times s_4 \bmod p$$

in the stated order, and lastly calculates

$$t=t_j \times m_j$$

for a value j chosen from a set of positive integers $\{1, 2, \dots, n\}$.

30. The method of Claim 26,

wherein each transformation process includes a coefficient group calculation process and transformation subprocesses,

4 performed following the coefficient group calculation process,
5 each for transforming a separate one of the object equations,
6 wherein in the coefficient group calculation process

7 (a) m nonzero coefficients are chosen by taking one nonzero
8 coefficient from each of the pivotal equation and the object
9 equations, each combination of $(m-1)$ of the chosen nonzero
10 coefficients is multiplied, and the m multiplication results are
11 set into the first coefficient group, m being a positive integer
12 no smaller than 2, and

13 (b) each of a constant and n coefficients in the pivotal
14 equation is multiplied by a multiplication result in the first
15 coefficient group for a combination of nonzero coefficients that
16 does not include a nonzero coefficient chosen from the pivotal
17 equation, and $n+1$ values obtained by the multiplications are set
18 into the second coefficient group,

19 wherein in each of the transformation subprocesses following
20 the coefficient group calculation process

21 (a) a nonzero coefficient chosen from the object equation in
22 the coefficient group calculation process is changed to 0 in the
23 object equation, and

24 (b) each of a constant and n coefficients in the object
25 equation is multiplied by a multiplication result in the first
26 coefficient group for a combination of nonzero coefficients that
27 does not include the nonzero coefficient chosen from the object

equation, and the $n+1$ values in the second coefficient group are subtracted respectively from the $n+1$ multiplication results.

31. The method of Claim 30,

wherein when the diagonal elements of the coefficient matrix C are denoted by m_i ($i=1,2,\dots,n$) and the inverses of the diagonal elements m_i ($i=1,2,\dots,n$) in the finite field $GF(p)$ are denoted by I_i ($i=1,2,\dots,n$), the diagonal element inverting step includes

(a) a multiplying substep for computing

$$t_i = \prod_{k=1}^n m_k \text{ (except } m_i) \text{ mod } p \text{ (} i=1,2,\dots,n \text{)}$$

and

$$t = \prod_{k=1}^n m_k \text{ mod } p$$

(b) a first inverting substep for computing

$$u = 1/t \text{ mod } p$$

and

(c) a second inverting substep for computing

$$I_i = u \times t_i \text{ mod } p \text{ (} i=1,2,\dots,n \text{)}$$

to find the inverses I_i ($i=1,2,\dots,n$).

32. The method of Claim 31,

wherein the multiplying substep calculates

3 $s_1 = m_1 \times m_2 \bmod p$
4 $s_2 = s_1 \times m_3 \bmod p$
5 \vdots
6 $s_{n-3} = s_{n-4} \times m_{n-2} \bmod p$
7 in the stated order, then calculates

8 $t_n = s_{n-3} \times m_{n-1} \bmod p$
9 $t_{n-1} = s_{n-3} \times m_n \bmod p$
10 $s_n = m_{n-1} \times m_n \bmod p$
11 $t_{n-2} = s_{n-4} \times s_n \bmod p$
12 $s_{n-1} = m_{n-2} \times s_n \bmod p$
13 $t_{n-3} = s_{n-5} \times s_{n-1} \bmod p$
14 $s_{n-2} = m_{n-3} \times s_{n-1} \bmod p$
15 $t_{n-4} = s_{n-6} \times s_{n-2} \bmod p$
16 \vdots
17 $s_5 = m_4 \times s_6 \bmod p$
18 $t_3 = s_1 \times s_5 \bmod p$
19 $s_4 = m_3 \times s_5 \bmod p$
20 $t_2 = m_1 \times s_4 \bmod p$
21 $t_1 = m_2 \times s_4 \bmod p$

22 in the stated order, and lastly calculates

23 $t = t_j \times m_j$

24 for a value j chosen from a set of positive integers
25 $\{1, 2, \dots, n\}$.